TRACE VELOCITIES AND WAVEFRONT OF A ROTATING LIGHT BEAM

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***Abstract**: At time t = 0, a pencil thin laser is oriented horizontally. It starts to rotate above an infinite plane P at a constant rate of rotation. The point of first impact is found and shown to be a point of singularity for velocity of the trace on P. From this point two traces of the laser beam travel in opposite directions. A mathematical model is constructed in six phases. These include a physical explanation phase.*

Key Words: modelling phases, Archimedes' spiral, curved light beam, wave front

1.0 Introduction

Picking a general problem often requires a subsequent selection of specific cases. When the question involves possible speeds of shadows, the challenge is to find an interesting configuration that has elegant expressions for shadow velocities. The configuration studied below also has a surprising result that infinite velocities are possible even near the observer – if conditions are right.

In addition, the configuration selected below shows that sometimes the original question can be rephrased to an equivalent, more relevant, question which is much easier to replicate in everyday life.

2.0 Phase I: Defining the Configuration

A pencil-beam laser pointer is rotating in a vertical plane at a rate of δ degrees/sec about a fixed point, located at a height, h, above an infinite horizontal plane, P, perpendicular to the plane of rotation. See Fig. 1. Cartesian coordinates (X,Y) are defined, with origin (0,0) on P and directly below the point of rotation. The X-axis is at the interesection of P and the plane of rotation. The Y-axis passes through (0,0) and the point (0,h).

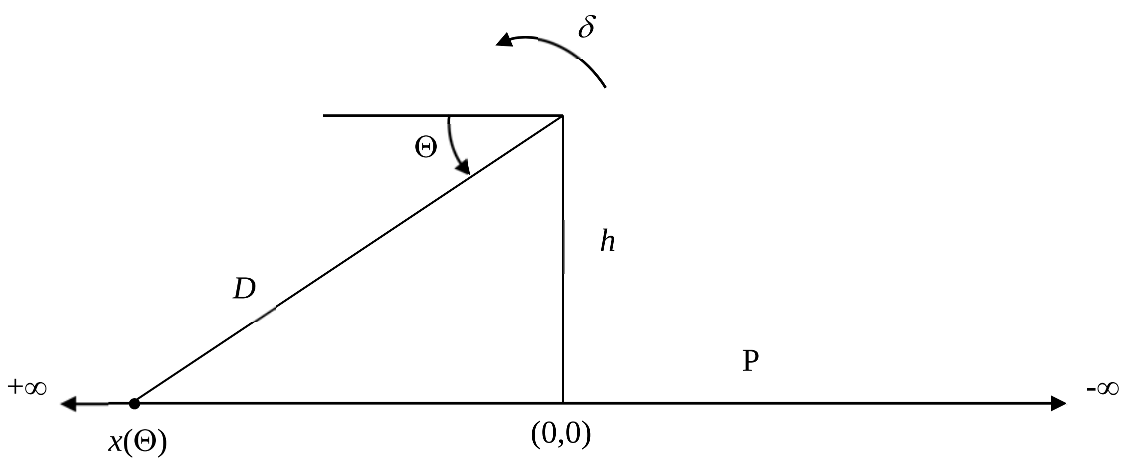

Fig.1 Parameters of the Configuration

At time t = 0, the laser instantaneously is in a horizontal position, sending photons along a parallel to the plane P. As t increases, the rotating laser produces a set of instantaneous „pencil-thin light beams" for each angle traversed. The intersection of P and the plane of rotation defines an X-axis with origin (0,0) directly below the point of rotation. The notation x(Θ) denotes the point that makes an angle Θ with the point of rotation, (0,h), as shown in Fig. 1. In other words, the photons that hit P at x(Θ) are those photons that left the laser when it was at angle Θ with respect to the horizontal through (0,h).

2.1 Parameters of Interest

The following have been determined below

a) the curve, C(t, Θ) that the photons from the rotating laser form at a time t,
b) t(Θ) = time at which the curve C(t, Θ) impacts point x(Θ),
c) the velocities, v(Θ) of the light beams as it „runs“ along the plane P. Of course the photons do not travel along P, but they impact P at different times. Hence their trace (or, „spot-light“) travels at some velocity, v(Θ) along the X-axis, where v(Θ) is the velocity of point x(Θ),
d) the point of first impact on P. This point is on the X-axis, and corresponds to a unique angle, denoted by Θ_m in Fig. 2 and Fig. 3, below.

3.0 Further Phases of the Modeling Activities

Defining the configuration just precises further the problem to be modelled. The phases of the further modelling where not mutually exclusive. They intermingled between qualitative/quantitative and between physical/mathematical. Sometimes one of these came in the forefront and sometimes another. They interchanged “as needed” without any specific order or constraints. This back and forth interchange lead to the full results stated herein. It indicates that the formal verbal description cannot be an accurate description of the modelling process, just a summary.

3.1 Outline of the Modelling Activities

After the configuration was defined, the modelling proceeded in a non-linear fashion. I.e., it skipped between phases. This occurred because the problem configuration was definitively „non-text book“, and each bit of insight gained in one phase called out questions about another phase. Following this inner process interactively lead to the full range of results herein.

E.g., the trace velocities on P were the initial property of interest, but this effort was „put on the backburner“ because no clear method for doing was immediately apparent. Next, a qualitative analysis graphically determined the curve C(t, Θ) based on very simple assumptions, and basically without any mathematics. The result showed that C(t, Θ) is

a) a universal curve (see Fig. 4),
b) the angle Θ works very well as an independent variable for the problem at hand,
c) there is a point of first impact in a finite portion on plane P.

The later insight was quickly „explained“ by physical reasoning. So the effort focused on finding quantitatively the point of first impact. With this goal in mind, the expressions for time t(Θ) of impact and for the angle Θ_m of initial impact were easily found.

A little reflection on the above modelling procedure shows that formal verbal descriptions as in this paper are very different than the sequence in which the model was actually built.

3.2 Phase II: Assumptions

The selection of assumptions is most important in constructing a model. Some assumptions lead to good results. Some others do not. E.g., in the case above, assuming that the speed of light is infinity leads to erroneous, unsatisfying results. Assuming that the photons travel at a finite speed, in straight lines as in Isaac Newton’s (Euclidean) space, (i.e., not as in the actual, Albert Einstein’s, space) gives good and interesting results, as shown below.

3.2.1 The Selected Assumptions

“Keep things as simple as possible, but not simpler.”, Albert Einstein

This activity too intermingled with the quantitative phase, and resulted in the following:

a) speed of light is a finite constant c,
b) light travels in straight lines according to Euclidean geometry,
c) the pencil-thin laser beams from the rotating laser are not divergent, i.e., can continue to infinity without spreading due to diffraction,
d) the photons travel in an unabsorbing medium.

3.3 Pase III: Solving for the Unknowns

Using the geometry of Fig. 1, the relevant geometrical parameters are

$$x(\Theta) = h\cot\Theta \qquad \text{and} \qquad D = h/\sin\Theta .$$

Clearly, the time t(Θ) is the sum of two times: the time for the laser to rotate to angle Θ, and the time for the photons to travel from their point of origin at (0,h) to the plane P, i.e., to the point x(Θ) on the X-axis. This gives

$$t(\Theta) = \frac{\Theta}{\partial} + \frac{h}{c\sin\Theta} .$$

To obtain the trace velocity at x(Θ), use the chain rule and implicit differentiation to get

$$v(\Theta) = \frac{\mathrm{d}x(\Theta)}{\mathrm{d}t} = \frac{\mathrm{d}x(\Theta)}{\mathrm{d}\Theta}\cdot\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \frac{\mathrm{d}x(\Theta)}{\mathrm{d}\Theta} \Big/ \frac{\mathrm{d}t}{\mathrm{d}\Theta} .$$

Now substitute into the above the following,

$$\frac{\mathrm{d}t}{\mathrm{d}\Theta} = \frac{1}{\partial} - \frac{K\cos\Theta}{\sin^2\Theta} = \frac{\sin^2\Theta - K\partial\cos\Theta}{\partial\sin^2\Theta}, \qquad \text{where} \qquad K = h/c$$

$$\frac{\mathrm{d}x(\Theta)}{\mathrm{d}\Theta} = -\frac{h}{\sin^2\Theta} .$$

This gives the velocity v(Θ) at x(Θ) as

$$v(\Theta) = \frac{-h\partial}{\sin^2\Theta - K\partial\cos\Theta} .$$

The curve of the wavefront at a given equal to a portion of the Archimedes' spiral, given by

C(t, Θ) = c (t - Θ/δ), where 0 < Θ < t δ

3.4 Phase IV: Descriptive Analyses

3.4.1 Graph of the Trace Velocities

The expression for the velocities has a singularity at Θ_m as indicated in Fig. 2 where Θ_m is the solution to the equation, dt(Θ)/d(Θ) = 0. I.e., the singularity occurs at the angle of first impact. The question of how and why the singularity occurs at all, and why it occurs at the angle of first impact is answered in the next section. The positive portion of the figure indicates that the spotlight is traveling in the positive direction (to the left of the reader). The negative velocities indicate that the spotlight is traveling to the right.

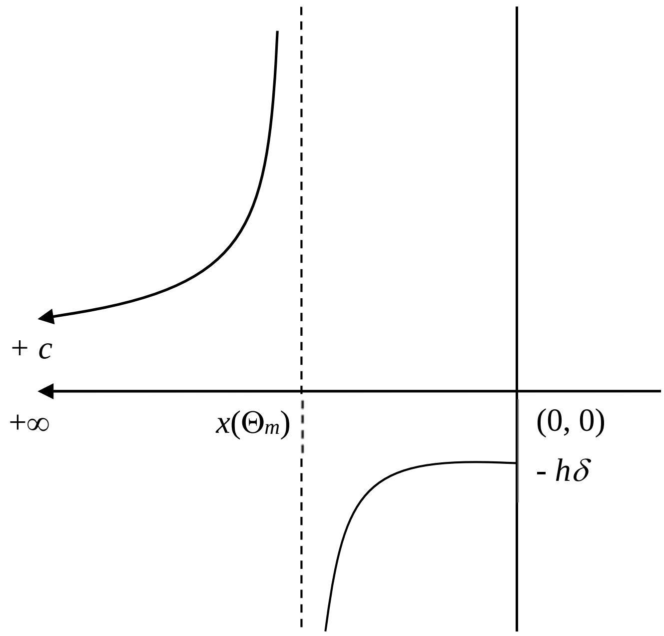

Fig. 2 Trace Velocities on the X-Axis of P

To draw the graph in Fig.2, the following limits were used

$$\lim_{\Theta \to 90} v(\Theta) = -h\partial$$

$$\lim_{\Theta \to 0^-} v(\Theta) = c$$

$$\lim_{\Theta \to \Theta_m^+} v(\Theta) = -\infty$$

$$\lim_{\Theta \to \Theta_m^-} v(\Theta) = +\infty$$

3.4.2 The Angle of First Impact

This angle is the solution of the equation dt(Θ)/d(Θ) = 0 It can be determined graphically as indicated in Fig. 3.

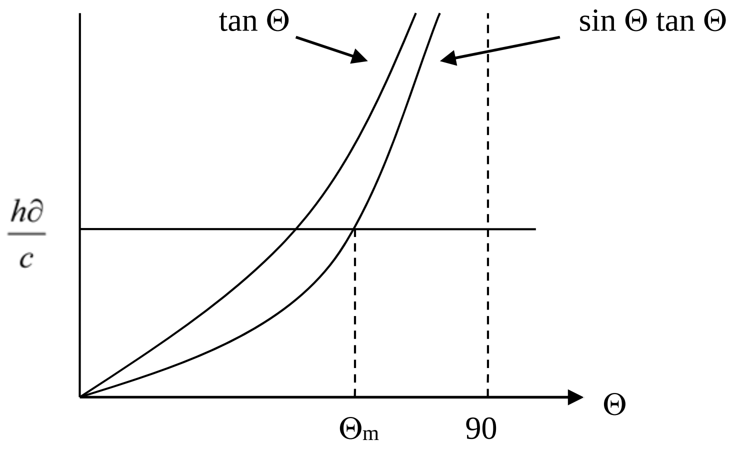

Fig. 3 Angle of first Impact on P

3.4.3 A Universal Graph for the Wave Front

For any given time $t > 0$, $C(t, \Theta)$ can be obtained graphically without any formal equation. This is shown in Fig. 4 by means of polar coordinates.

The angles between the rays are equal to some angle, A. The rotating laser rotates through A in a time A/δ. During this time any photon that is already enroute travels a distance

$L = cA/\delta$.

Fig.4 is a universal graph where the difference in radii between two consecutive circular arcs of the polar coordinate system is L, if and only if the angle between two consecutive rays is A.

3.4.4 A Rotating Archimedes' Spiral

At a fixed time, t the pattern of the photons in the plane of rotation is described by a segment of an Archimedes' spiral, as given by $C(t, \Theta)$. As time increases the photons move onward along the rays of the polar coordinates, and the Archimedes' spiral rotates counter clockwise

3.5 Phase V: Physical Analysis

In Fig. 4, the wave front $C(t, \Theta)$ first impacts on P at a point where its tangent is horizontal. The rate of change $dC(t, \Theta)/dx = 0$ at Θ_m.. This implies instantaneously infinite velocities at $x(\Theta_m)$, just as Fig. 2 indicates.

4.0 Conclusion

No references to similar questions or results have been found, but artist pictures of rotating pulsars show curved light beams. This indicates that the phenomena studied in Fig. 4 is at least partially known. However, the first impact and velocity results are probably original.

4.1 Phase VI: Identified Extensions

a) Solutions have been obtained explicitly in the first quadrant, $0 < \Theta < 90$ degrees. Extending them to $0 < \Theta < 180$ degrees is no problem because the trigonometric functions change appropriately in the second quadrant. For $180 < \Theta < (360 + \Theta_m)$ there is no new activity on P. Of course, thanks to assumptions c) and d) the two spotlights continue traveling to their respective infinities. Going over $(360 + \Theta_m)$ degrees will require a slight change of notation because then multiple traces will be traveling in both the plus and minus directions along the X-axis.

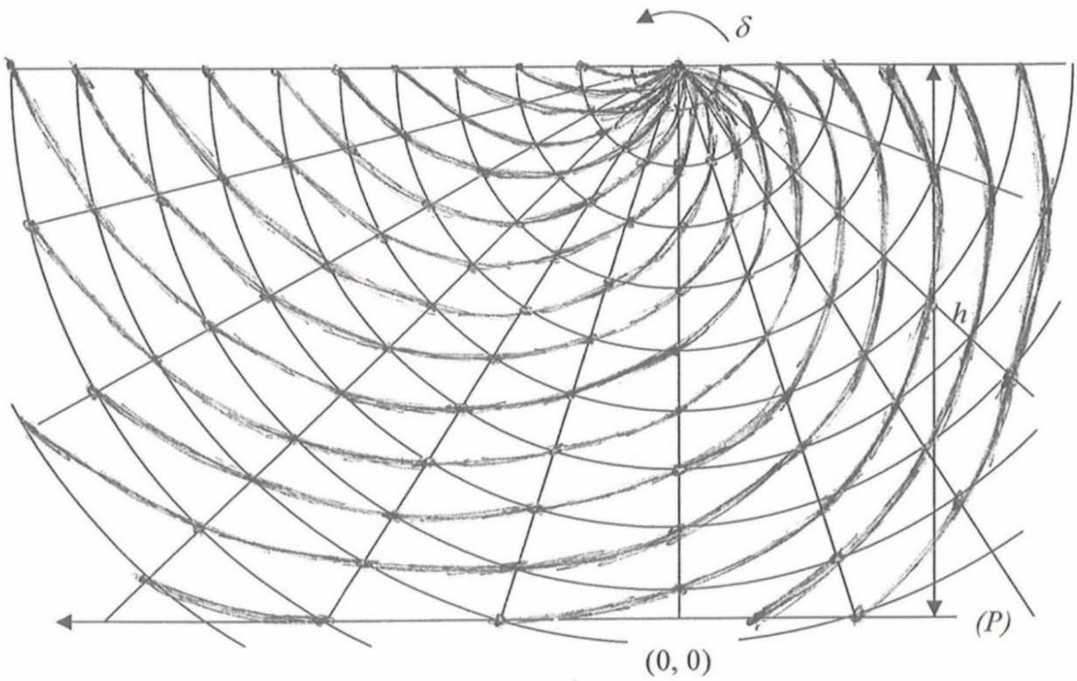

Fig 4 Trace of Light Wavefront in Space
(Universal Graph)

b) A sphere S can replace P in Fig. 1 and the same modeling applied. Yes, there is a point of first impact on S. Yes, there are two instaneously infinite velocites at that point, and they travel on S in opposite directions.

4.2 Recommended Experiment

For over a decade the technology of time measurement been brought down to the order attoseconds (10^{-18}). It might be possible to devise an experiment with a powerful laser, to catch the point of its first impact on Earth, and to measure trace velocities in the vicinity of that point.

4.3 A Speculation on Gravitons

The Einstein,Thirring,Lense effect indicates that a spinning mass drags space along with it, just as a spinning circle on a rubber fabric will distort the fabric, with a twisting force. This effect has been noted for satelites, where it results in a drag of about 30 mili-arcseconds/yr. (Eckstein, 2013).

It is of interest to speculate if the „wavefront“ of a stream of gravitrons issueing from a spinning mass also bends in a simillar fashion to the stream of photons as in Fig. 4. If they do then in the neighborhood of the Solar System, this bending will be much smaller than the other warping effect predicted by general relativity, and hence not obviously distinguishable from it. In fact, if both photons and graviton wavefronts from spinning objects bend in the „same“ way, the gravitational pull will still be as predicted by the general relativity, because analagously to the photons depicted in Fig. 4, the gravitrons would be traveling in a different direction from their wave front.

There might be two sources of the gravitron bending; one from the spinning matter that is being observed, and the other from the warping drag due to the spin of the Earth itself.

Honor of Albert Einstein

Since it is still very close to the 100th. anniversary (November 2015) of Alfred Einstein's theory of general relativity/gravity, it is fitting to pay him homage by mentioning several even trivial points of interest that the above results somehow relate to him directly.

a) When Albert Einstein was a teenager in high school, he had his first (known) thought experiment of traveling on the front of a light wave. It seems as if everybody (including the author) imagines that light wave to be the front of a stationary light beam. However, after seeing the curve of the photons at time t, a definite claim can be made that Albert Einstein imagined the curves in the universal graph of figure 4 above, especially because it is absolutely easy to construct without any math other than arithmetic and and the concept of constant linear and rotational speeds.

b) When a rotating flashlight or a beam from a lighthouse is seen at night it appears as a straight line sweeping through a misty night. Ditto a sweeping WWII searchlight beam. As shown above, the apparent „straight line“ is actually a segment of a rotating Archimedes‘ spiral. The illusion of a straight line occurs because of the minute distances involved in our field of perception. The following observation by Albert Einstein,

"Reality is merely an illusion, albeit a very persistent one.", seems appropriate.

Acknowledgement

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Reference

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